Let $a, b \in \mathbb{N}$; $1 < a < b$

Let $c = \log_a(b)$ rounded up to integer

$\Rightarrow \underline{1} < c < b \quad \wedge \quad a^c \geq b$

Define function $u: \mathbb{N}^{>0} \times \mathbb{N}^{>0} \times \mathbb{N}^{\geq 0} \to \mathbb{N}^{>0}$ recursively:

$$u(a, x, \emptyset) := x$$

$$u(a, x, k+1) := a^{u(a,x,k)}$$

(Then as a special case we have $u(a, a, k) = a[4](k+1)$)
(Also $u(a, a[4]m, k) = a[4](k+m)$)

Now let $x, y, k \in \mathbb{N}$ such that $x > 0$, $y > 0$, $k \geq 0$, and

$x \geq c * (y + k)$ (i)

Then the following can be proved:

(ii) $\forall l \in \mathbb{N}, \; 0 \leq l \leq k \; : \; u(a, x, l) \geq c * (u(b, y, l) + k - l)$

Proof by induction on l:

$l = \emptyset$ directly gives (i)

Now assume (ii) to be true for l, then we wish to prove that

$l+1 \leq k \Rightarrow u(a, x, l+1) \geq c * (u(b, y, l+1) + k - (l+1))$

$$u(a,x,l+1) = a^{u(a,x,l)} \geq b^{(u(b,y,l)+k-l)}$$

$$= b * \left(b^{(u(b,y,l)+k-(l+1))} \right) \geq c * \left(b^{(u(b,y,l)+k-(l+1))} \right)$$

$$\geq c * \left(b^{u(b,y,l)} + k - (l+1) \right) = c * \left(u(b,y,l+1) + k - (l+1) \right)$$

This concludes the prove of (ii).

From (ii) follows:

$$x \geq c * (y+k) \Rightarrow u(a,x,k) \geq c * u(b,y,k)$$

$$\Rightarrow u(a,x,k+1) \geq u(b,y,k+1)$$

As special case:

$$a[4]m \geq c * (b+k) \Rightarrow u(a, a[4]m, k+1) \geq u(b, b, k+1)$$

$$\Rightarrow a[4](k+m+1) \geq b[4](k+2)$$